WARMUP! ☺

Describe the process you would use if you were to solve the following problem:

- Write the equation of the parabola that passes through the point (-1, 2) and has a vertex of (4, -9).

Note: you are not being asked to solve the problem…

1. Identify information provided (vertex and a point)
2. Determine which form to start with (vertex form)
3. Plug in what you know (h, k, x, and y)
4. Solve for what is left (a)

Today you will:

- Write equations of quadratic functions given only points on the parabola
- Write quadratic equations to model data sets
- Practice using English to describe math processes and equations

Given the following points that are taken from a parabola, write the equation for the quadratic:

(1, 3), (2, 7), (3, 13), (4, 21) … hint, none are a vertex …

- Which form should we use for the equation?
  - Have no obviously useful info … so use Standard Form: $f(x) = ax^2 + bx + c$

- What do we know?
  - A bunch of x and y values (coordinate points)…

- Pick a point, plug in what we know (the x and y values) and see what we are left with…
  - Let's go with (1, 3)
  - $y = ax^2 + bx + c \longrightarrow 3 = a(1)^2 + b(1) + c \longrightarrow 3 = a + b + c$

- So we have an equation with 3 variables (a, b and c) … can we solve that? What if we had 3 of these?

- Pick 2 other points and do the same again with them:
  - (2, 7) $\longrightarrow 7 = a(2)^2 + b(2) + c \longrightarrow 7 = 4a + 2b + c$
  - (3, 13) $\longrightarrow 13 = a(3)^2 + b(3) + c \longrightarrow 13 = 9a + 3b + c$

- So we now have:
  - $a + b + c = 3$
  - $4a + 2b + c = 7$
  - $9a + 3b + c = 13$

- So we now have:

$$a + b + c = 3$$
$$4a + 2b + c = 7$$
$$9a + 3b + c = 13$$

- Use either elimination or substitution to solve … let's do substitution:

$a + b + c = 3$ so $c = -a - b + 3$

$4a + 2b + (-a - b + 3) = 7$ ← *substitute back into 2nd equation*

$$3a + b = 4$$

$9a + 3b + (-a - b + 3) = 13$ ← *substitute back into 3rd equation*

$$8a + 2b = 10$$
$$4a + b = 5$$

$$-3a - b = -4$$
$$4a + b = 5$$

$$a = 1$$
$b = 1$ $\quad from\ 3a + b = 4$

$c = 1$ $\quad from\ a + b + c = 3$

So $y = x^2 + x + 1$

So … what did we just do?

Exactly what we did the day before…

1. Identify information provided (a bunch of points but no vertex or intercept info)
2. Determine which form to start with (standard form)
3. Plug in what you know (pick a point plug in the *x* and *y …* repeat with 2 more points)
4. Now have a system of 3 equations, solve for what is left (a, b, c) using elimination/substitution!

How do you tell if a set of points are parabolic?

If the points are evenly spaced (the differences between the *x* values are the same)

Then look at the <u>differences of the *y* values</u> … and the <u>differences of the differences</u>

1st differences — 2nd differences

(1, 3) (2, 7) (3, 13) (4, 21)

1st differences → 4 6 8

2nd differences → 2 2

If the 2nd differences are constant (the same), the points are parabolic.

By the way, how do you tell if a set of points are linear?

If the points are evenly spaced (the differences between the *x* values are the same)

Then look at the differences of the *y* values ...

1st differences

(1, 1) (2, 4) (3, 7) (4, 10)

1st differences → 3 3 3

If the 1st differences are constant (the same), the points are linear.

Example 3 (from the book)

Time, *t*	Height, *h*
10	26,900
15	29,025
20	30,600
25	31,625
30	32,100
35	32,025
40	31,400

NASA can create a weightless environment by flying a plane in parabolic paths. The table shows heights *h* (in feet) of a plane *t* seconds after starting the flight path. After about 20.8 seconds, passengers begin to experience a weightless environment. Write and evaluate a function to approximate the height at which this occurs.

SOLUTION

Step 1 The input values are equally spaced. So, analyze the differences in the outputs to determine what type of function you can use to model the data.

$h(10)$	$h(15)$	$h(20)$	$h(25)$	$h(30)$	$h(35)$	$h(40)$
26,900	29,025	30,600	31,625	32,100	32,025	31,400

First differences: 2125, 1575, 1025, 475, −75, −625

Second differences: −550, −550, −550, −550, −550

Because the second differences are constant, you can model the data with a quadratic function.

Step 2 Write a quadratic function of the form $h(t) = at^2 + bt + c$ that models the data. Use any three points (t, h) from the table to write a system of equations.

Use (10, 26,900):	$100a + 10b + c = 26{,}900$	Equation 1
Use (20, 30,600):	$400a + 20b + c = 30{,}600$	Equation 2
Use (30, 32,100):	$900a + 30b + c = 32{,}100$	Equation 3

Use the elimination method to solve the system.

Subtract Equation 1 from Equation 2. → $300a + 10b = 3700$ — New Equation 1

Subtract Equation 1 from Equation 3. → $800a + 20b = 5200$ — New Equation 2

$200a = -2200$ — Subtract 2 times new Equation 1 from new Equation 2.

$a = -11$ — Solve for a.

$b = 700$ — Substitute into new Equation 1 to find b.

$c = 21{,}000$ — Substitute into Equation 1 to find c.

The data can be modeled by the function $h(t) = -11t^2 + 700t + 21{,}000$.

Step 3 Evaluate the function when $t = 20.8$.

$$h(20.8) = -11(20.8)^2 + 700(20.8) + 21{,}000 = 30{,}800.96$$

▶ Passengers begin to experience a weightless environment at about 30,800 feet.

Homework:

- Pg 81, #23-27